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As per CBCS

B.Sc Mathematics – I

Differential Equations

I - Semester

REFERENCE BOOK

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SPECTRUM®

Differential Equations

As per Choice Based Credit System (CBCS)

I- B. Sc / I - Semester

REFERENCE BOOK

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I - Year / I - *Semester* as per CBCS

Differential Equations

Course Outcomes

After successful completion of this course, the student will be able to;

1. Solve linear differential equations
2. Convert non exact homogeneous equations to exact differential equations by using integrating factors.
3. Know the methods of finding solutions of differential equations of the first order but not of the first degree.
4. Solve higher-order linear differential equations, both homogeneous and non homogeneous, with constant coefficients.
5. Understand the concept and apply appropriate methods for solving differential equations.

Unit-I: Differential Equations of first order and first degree

Linear Differential Equations; Differential equations reducible to linear form; Exact differential equations; Integrating factors; Change of variables.

Unit-II: Orthoginal Trajectories Differential Equations of first order but not of the first degree

Equations solvable for p; Equations solvable for y; Equations solvable for x; Equations that do not contain x (or y); Equations homogeneous in x and y; Equations of the first degree in x and y – Clairaut's Equation.

Unit-III: Higher order linear differential equations-I

Solution of homogeneous linear differential equations of order n with constant coefficients; Solution of the non-homogeneous linear differential equations with constant coefficients by means of polynomial operators. General Solution of $f(D)y = 0$.

General Solution of $f(D)y = Q$ when Q is a function of $x, \frac{1}{f(D)}$ is expressed as partial fractions.

P.I. of $f(D)y = Q$ when $Q = be^{ax}$, P.I. of $f(D)y = Q$ when Q is b sinax or b cos ax.

Unit-IV:Higher Order Linear Differential Equations-II

Solution of the non-homogeneous linear differential equations with constant coefficients.

P.I. of $f(D)\ y = Q$ when $Q = bx^k$

P.I. of $f(D)\ y = Q$ when $Q = e^{ax}V$, where V is a function of x.

P.I. of $f(D)\ y = Q$ when $Q = xV$, where V is a function of x.

P.I. of $f(D)\ y = Q$ when $Q = x^mV$, when V is a function of x.

Unit-V: Higher Order Linear Differential Equations-III

Method of variation of parameters; Linear Differential Equations with non-constant coefficients; The Cauchy-Euler Equation, Legendre's linear equations, miscellaneous differential equations.

Content

Differential Equations

Chapter-1: Introduction

1.0 OBJECTIVES

1. Identify origins and applications of differential equations.
2. Describe what is meant by solutions of Differential Equations.
3. Discuss what is meant by Initial-Value Problems, Existence and Uniqueness of solutions.
4. Recognize order and degree and classify ordinary di?erential equations.
5. Evaluate first order differential equations including separable, homogeneous.

1.1 INTRODUCTION

The Mathematical model of differential equation represents in solving the problems faced not only in Physics, Chemistry, Economics and many engineering branches but also in Psychology and Biology. The study of differential equations playing crucial role in mathematics, and in view of the applications, in other sciences too. The role of differential equations is unique in suggesting solutions to many difficult problems in other sciences quickly and easily.

If we want to solve an engineering problem (usually of a physical nature), we first have to formulate the problem as a mathematical expression in terms of variables, functions, and equations. Such an expression is known as a **mathematical model** of the given problem. The process of setting up a model, solving it mathematically, and interpreting the result in physical or other terms is called **mathematical modeling** or, briefly, **modeling**.

A model is very often an equation containing derivatives of an unknown function. Such a model is called a **differential equation**. We then want to find a solution (a function that satisfies the equation), explore its properties, graph it, find values of it, and interpret it in physical terms so that we can understand the behavior of the physical system in our given problem. However, before we can turn to methods of solution, we must first define some basic concepts needed throughout this chapter.

The study of differential equations and their advancement was due to great mathematicians like Newton, John Bernoulli, Joseph-Louis Lagrange, Leonhard Euler, Laplace, Fourier, Abel, Gauss, D'Alembert, Leibniz and others. Renowned mathematician Leonhard Euler contributed many of the methods of solving differential equations.

Let us observe how some of the various differential equations are useful in different branches

1. The differential equation $\frac{dN}{dt} = -N\alpha$ represents the decay of radio active elements.
2. The differential equation $xy'' + y' + xy = 0$ arises in the field of mechanics, electrictiy, heat, aerodynamics and so on.
3. The differential equation $F = M\frac{dv}{dt} - V\frac{dm}{dt}$ represents the thrust on a rocket resulting from ejection of gases.
4. In quantam mechanics, the partial differential equation $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$ is used to find the wave function. This is known as scrodinger wave equation.
5. The differential equation $\frac{dNi}{dt} = kN_i(N - N_i)T$ is used by medical scientists to find the number of infected persons at any time to control the spread of infectious diseases.
6. In medical sciences, the rate of growth of tumors is determined by using the differential equation, $\frac{dv}{dt} = ke^{-at}V$.
7. The partial differntial equation, $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$ is called as Laplace equation arises in the study of heat, electricity, gravitation, aerodynamics and so on.
8. The differntial equation $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E_0 \cos\omega t$ represents the oscillation of electricity in circuits containing inductors, resistors and capacitors.

1.2 DIFFERENTIAL EQUATIONS

1.2.1 Basic Concepts

In finding the solutions of differential equations we can integrate many functions. Some of Integrants are given below.

1) $\int x^n dx = \frac{x^{n+1}}{x+1} + c, n \neq -1$

2) $\int \frac{1}{x} dx = \log|x| + c$

3) (a) $\int e^x dx = e^x + c$

(b) $\int \log x \, dx = x \log x - x + c$

4) $\int a^x dx = \frac{a^x}{\log_e a} + c$

5) $\int \sin x \, dx = -\cos x + c$

6) $\int \cos x \, dx = \sin x + c$

7) $\int \sec^2 x \, dx = \text{Tan } x + c$

8) $\int \text{cosec}^2 x dx = -\cot x + c$

9) $\int \sec x \, \text{Tan } x \, dx = \sec x + c$

10) $\int \cot x \, \text{cosec } x \, dx = -\text{cosec } x + c$

11) $\int \tan x\,dx = -\log|\cos x| + c$ (or) $\log|\sec x| + c$

12) $\int \cot x\,dx = \log|\sin x| + c$

13) $\int \sec x\,dx = \begin{cases} \log|\sec x + \tan x| + c \text{ (or)} \\ \log\tan(\pi/4 + x/2) \end{cases}$

14) $\int \operatorname{cosec} x\,dx = \begin{matrix} \log|\operatorname{cosec} x - \cot x| + c \text{ (or)} \\ |\log\tan x/2| + c \end{matrix}$

15) $\int \sinh x\,dx = \cosh x + c$

16) $\int \cosh x\,dx = \sinh x + c$

17) $\int \operatorname{sech}^2 x = \tanh x + c$

18) $\int \operatorname{cosech}^2 x\,dx = -\coth x + c$

19) $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + c$

20) $\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + c$

21) $\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a} + c$ (or) $-\cos^{-1}\frac{x}{a} + c$

22) $\int \frac{-1}{\sqrt{a^2 - x^2}}dx = -\sin^{-1}\frac{x}{a} + c$ (or) $\cos^{-1}\frac{x}{a} + c$

23) $\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

24) $\int \frac{-1}{a^2 + x^2}dx = \frac{1}{a}\cot^{-1}\frac{x}{a} + c$

25) $\int_x \frac{1}{\sqrt{x^2 - a^2}}dx = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

26) $\int \frac{-1}{\sqrt{x^2 - a^2}}dx = \frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a} + c$

27) $\int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c, x > a$

28) $\int \frac{1}{a^2 - x^2}dx = \frac{1}{2a}\log\left|\frac{a + x}{a - x}\right|, x < a = \frac{1}{a}\tanh^{-1}\frac{x}{a} + c$

29) $\int \frac{1}{\sqrt{a^2 + x^2}}dx = \sinh^{-1}\frac{x}{a} + c$ (or) $\log\left|x + \sqrt{(x^2 + a^2)}\right| + c$

30) $\int \frac{1}{\sqrt{x^2 - a^2}}dx = \cosh^{-1}\frac{x}{a} + c$ (or) $\log\left|x + \sqrt{x^2 - a^2}\right| + c$

31) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

32) $\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + c$

33) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

34) $\int \frac{f^1(x)}{f(x)}dx = \log|f(x)| + c$

35) $\int [f(x)]^n f^1(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c, n \neq -1$

3. $-x\cos x + 2\sin x + c_1 x + c_2$

4. $\frac{1}{4}(x-1)e^{2x} + c_1 x + c_2$

5. $c_1 + c_2 x - \frac{1}{2}(e^x \cos x)$

6.7 OUTCOMES

- Fnd the complete solution of a nonhomogeneous differential equation with constant coefficients by the method of undetermined coefficients.
- Find the complete solution of a differential equation with constant coefficients by variation of parameters.

6.8 MULTIPLE CHOICE QUESTIONS

1. Which one of the following is Cauchy-Euler's Differential Equation ( )

(a) $\frac{d^n y}{dx^n} + \frac{d^{n-1} y}{dx^{n-1}} + \ldots + \frac{dy}{dx} + y = Q(x)$ (b) $a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + \frac{dy}{dx} + y = Q(x)$

(c) $a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1 x \frac{dy}{dx} + a_0 y = Q(x)$

(d) $k_n (ax+b)^n \frac{d^n y}{dx^n} + k_{n-1} (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + k_n y = Q(x)$

2. Using the method of variation of parameters for the particular solution to the D.E. is

$y'' + 4y = \frac{3}{\sin 2x}, 0 < x < \pi/2$ ( )

(a) $\frac{3}{4}\sin 2x \log \sin 2x + \frac{3}{2} x \cos 2x$ (b) $\frac{3}{4}\cos 2x \log \cos 2x + \frac{3}{2} x \sin 2x$

(c) $\frac{3}{4}\sin 2x \log \sin 2x - \frac{3}{2} x \cos 2x$ (d) $\frac{3}{4}\cos 2x \log \cos 2x - \frac{3}{2} x \sin 2x$

3. The solution of $\frac{d^2 y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, if $P + Qx = 0$ is ( )

(a) $y = x$ (b) $y = e^x$ (c) $y = e^{-x}$ (d) $y = x^2$

4. The solution of $\frac{d^2 y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, if $1 - P + Q = 0$ is ( )

(a) $y = x$ (b) $y = x^2$ (c) $y = e^x$ (d) $y = e^{-x}$

5. If $y = x$ is a solution of $x^2 y'' + xy' - y = 0$, then the second linearly Independent solution is ( )

(a) $\frac{1}{x^2}$ (b) $\frac{1}{x}$ (c) e^x (d) x^2

6. If $y(x)=e^{4x}V(x)$ is a particular solution of the D.E.

$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=\left(2x+11x^{10}+21x^{20}\right)e^{4x}$ then V(x) is **[IIT-JAM-2011]** ( )

(a) $c_1x+c_2+\frac{x^3}{3}+\frac{x^{12}}{12}+\frac{x^{22}}{22}$ (b) $c_1x-c_2-\frac{x^3}{3}-\frac{x^{12}}{12}$

(c) $c_1x+c_2+\frac{x^3}{3}+\frac{x^{22}}{22}$ (d) None of these

7. Particular Integral of the D.E $y^{11}-4y^1+4y=\frac{e^{2x}}{x}$ is ( )

(a) $e^x(x+\log x)$ (b) $e^{2x}(-x-\log x)$ (c) $e^x(-x-\log x)$ (d) $e^{2x}(x+\log x)$

8. If the D.E. $y^{11}+Py^1+Qy=0$ and $2+2Px+x^2=0$ then one of the solutions of the ab are D.E. is ( )

(a) $y=e^x$ (b) $y=e^{-x}$ (c) $y=x$ (d) $y=x^2$

9. One of the solutions of the D.E. $x^2y^{11}+xy^1-y=0$ is ( )

(a) $x-\frac{1}{x}$ (b) $x^2+\frac{1}{x}$ (c) $x+\frac{1}{x}$ (d) $x^2-\frac{1}{x}$

10. The particular Integral of the D.E. $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$ is Au +BV, by variation of parameters method, the value of 'A' is ( )

(a) $\int\frac{-VRdx}{uV_1-Vu_1}$ (b) $\int\frac{uRdx}{uV_1-Vu_1}$ (c) $\int\frac{VRdx}{uV_1-Vu_1}$ (d) $\int\frac{-VRdx}{uV_1+Vu_1}$

11. By using variation of parameters method, the value of C.F. of the $\left[(x-1)D^2-xD+1\right]+y=(x-1)^2$ is ( )

(a) $c_1e^{-x}-c_2x$ (b) $c_1e^{-x}+c_2x$ (c) $c_1e^x+c_2x^2$ (d) $c_1e^{-x}+c_2x^2$

12. Complementary function of the D.E. $(1-x)y^{11}+xy^1-y=2(x-1)^2e^{-x}$ is ( )

(a) $c_1x^2+c_2e^x$ (b) $c_1e^x+c_2x$ (c) $c_1x+c_2x^{-1}$ (d) $c_1x+c_2e^x$

13. The value of 'x' for the system of D.E. $\frac{dx}{dt}-7x+y=0$ and $\frac{dy}{dt}-2x-5y=0$ is ( )

(a) $e^{6t}(c_1\cos t+c_2\sin t)$ (b) $e^{-6t}(c_1\cos t+c_2\sin t)$

(c) $e^{6t}(c_1\sin t+t)$ (d) $e^{6t}(c_1\cos t+t)$

14. The value of 'y' for the system of Differential equations $2\frac{d^2y}{dx^2}-\frac{dz}{dx}-4y=2x$ and $2\frac{dy}{dx}+4\frac{dz}{dx}-3z=0$ is ( )

(a) $(c_1+c_2x)e^x+c_3e^{-3x/2}+\frac{x}{2}$ (b) $(c_1+c_2x)e^x+c_3e^{-3x/2}-\frac{x}{2}$

(c) $c_1e^x+(c_2+c_3x)e^{3x/2}+3x$ (d) None of these

15. Complementary function of the D.E. $x^2y^{11} - 6xy^1 + 12y = x$ is ( )

(a) $c_1e^x + c_2x^3$ (b) $c_1e^{-x} + c_2x^4$ (c) $c_1x^3 + c_2x^4$ (d) $c_1x^2 + c_2x^3$

16. Solution of the D.E. $x^2y^{11} - 3xy^1 + 4y = 0$ is ( )

(a) $(c_1 + c_2x^2 \log x)$ (b) $(c_1 + c_2e^x)x^2$

(c) $(c_1 + c_2 \log x)x$ (d) $(c_1 + c_2 \log x)x^2$

17. Solution of the D.E. $x^2y^{11} + xy^1 - 4y = 0$ is ( )

(a) $c_1x + c_2x^{-1}$ (b) $c_1x^2 + c_2x^{-2}$ (c) $c_1x^2 - c_2x^{-2}$ (d) $c_1x^2 + c_2x^{-1}$

18. Complementary function of the D.E $x^3y^{111} - 3x^2y^{11} + 6xy^1 - 6y = (\log x)^2$ is ( )

(a) $c_1e^x + c_2x + c_3x^2$ (b) $c_1e^{-x} + c_2x + c_3x^2$

(c) $c_1x + c_2x^2 + c_3x^3$ (d) $c_1x^x + c_2x^2 + c_3x^3$

19. Solution of the D.E. $(x+1)^2y^{11} - 4(x+1)y^1 + 6y = 0$ is ( )

(a) $c_1(x+1)^2 + c_2(x+1)^3$ (b) $c_1(x+1) + c_2(x+1)^2$

(c) $c_1(x+1) + c_2(x+1)^3$ (d) None of these

20. Solution of the D.E. $(x^2D^2 - 5xD + 8)y = 2x^3$ is ( )

(a) $c_1x + c_2x^2 - 2x^3$ (b) $c_1x^2 + c_2x^3 - 2x^3$

(c) $c_1x^2 + c_2x^4 - 2x^3$ (d) $c_1x^2 + c_2x^4 - 2x^3$

Answers

1. (c)	2. (c)	3. (a)	4. (d)	5. (b)
6. (a)	7. (b)	8. (d)	9. (c)	10. (a)
11. (b)	12. (d)	13. (a)	14. (b)	15. (c)
16. (d)	17. (b)	18. (c)	19. (a)	20. (d)

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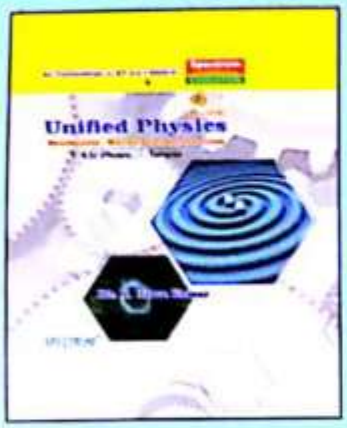

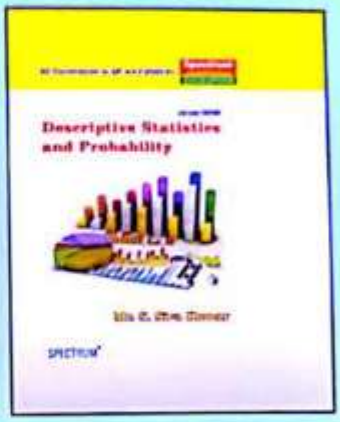

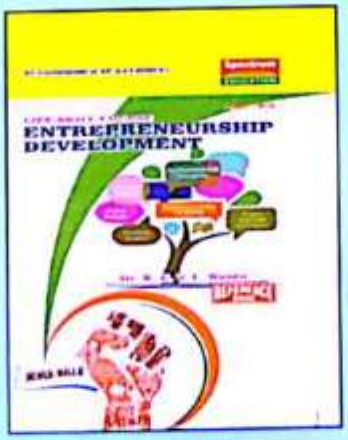

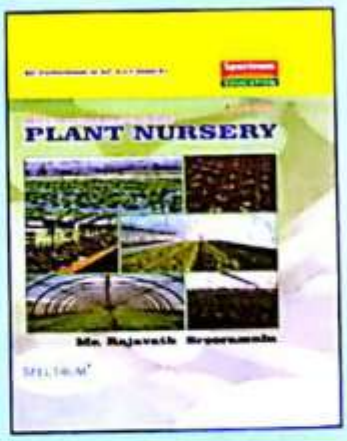

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